06-Bortle Calculation Design Document

Minimum Bortle Sky Class Estimator — Technical Reference (v2, fixed 150 mm aperture)

1. Purpose

The mBortle tool estimates, for a fixed 150 mm (6-inch) telescope and an average visual observer, the worst (highest-numbered, most light-polluted) Bortle sky class at which a given deep-sky object can still be detected or resolved. It is the inversion of a companion "minimum aperture" tool, which instead fixes the sky at Bortle 4 and solves for the required telescope size. Output is written back into a spreadsheet as an integer Bortle class (1–9) or "Err" when the object is not detectable in 150 mm under any real sky, or a resolution/data problem prevents an estimate.

2. What the Script Does

The tool reads a workbook of catalog objects, classifies each row by object type, routes it to one of three physics modules, and performs a discrete sweep over nine representative sky-brightness levels (Bortle 1, the darkest, down to Bortle 9, the brightest) to find the worst class that still passes a detection test. The first (highest-numbered) class that passes is recorded; if none pass, the cell is set to "Err."

Every "Err" result is yellow-highlighted with an attached cell comment explaining the specific cause — a missing input, a resolution limit that no sky can overcome, or a brightness/contrast shortfall even under the darkest sky modeled. Only the mBortle column is modified; all other columns (including a pre-existing minimum-aperture column, mApt) are left untouched.

3. Inputs Expected

One Excel row per object (or per pairing, for multiple star systems), with these columns:

- ID — object identifier/name
- Type — object classification code (see table below)
- Magnitude, Magnitude 2 — numeric or numeric-as-text; "Magnitude 2" values of 99, 99.0, or 0 are treated as null placeholders
- Size — angular size in arcminutes; either a single value or "AxB" (major×minor), in which case the geometric mean $\sqrt{(A \cdot B)}$ is used
- Surface Brightness — mag/arcsec², catalog average (may be "N/A" for types that do not need it)
- Separation — arcseconds, for double/multiple stars
- M Class — an overloaded field; for dark nebulae it carries the Lynds opacity class as "Opacity=N"; other codes in this column (globular morphology, cluster class) are ignored by this tool
- mBortle — the output column, populated by the tool

Type code	Object	Routing
AS	Asterism	Skipped entirely
Star	Single star	Point-source test on Magnitude
CS	Carbon star	Point-source test on mean of Magnitude/Magnitude 2 (if both present)
DS / Dbl / MS-x	Double/multiple star	Resolution gate, then point-source test on fainter component
OC	Open cluster	Point-source test on integrated Magnitude + 4.0
PN (<0.25′)	Small planetary nebula	Treated as a point source
PN (≥0.25′), GC	Large planetary nebula, globular cluster	Extended-object contrast test
DN	Dark nebula	Decrement-contrast test with Milky Way visibility limit

4. Values Calculated

A single output per in-scope row: mBortle, an integer 1–9 (or "Err"). 9 means detectable from an inner city; 1 means only the darkest rural skies will show it; Err means the 150 mm aperture cannot reach the object at any sky brightness, or required data/resolution is missing.

5. Physics and Equations

5.1 Shared frame

All three modules sweep over a fixed table of nine representative sky brightness values, anchored so that Bortle 4 = 21.0 mag/arcsec² (matching the companion aperture tool):

```
Bortle:      1     2     3     4     5     6    7      8     9
μ_sky (mag/arcsec²): 21.9  21.7  21.4  21.0  19.8  18.9 18.25  17.5  17.0
```

The sweep runs from Bortle 9 down to 1; the first class that passes the module's detection test is returned as mBortle.

5.2 Module 1 — Point sources (stars, doubles, carbon stars, open clusters, small PN)

```
m_lim(b) = 2.7 + 5·log₁₀(D) − 5·log₁₀(1.2) − 0.15 − ΔLP(b)
ΔLP(b) = 0.30 + 0.20 · (21.0 − μ_sky(b))
```

D = aperture (150 mm, fixed). The constant 2.7 is the classical telescopic limiting-magnitude zero point. The $5 \cdot \log_{10}(1.2)$ term subtracts a 20% aperture safety margin. 0.15 mag accounts for atmospheric extinction at a typical 45° target altitude. ΔLP(b) is the light-pollution penalty at Bortle class b, scaled from a Bortle-4 baseline of 0.30 mag at a rate of 0.20 mag per mag/arcsec² of sky brightness change — reflecting that telescopic point-source limits are only weakly sky-dependent once magnification darkens the field. mBortle is the highest b for which the object's magnitude m ≤ m_lim(b).

Double stars are additionally screened by the Rayleigh resolution criterion before any brightness test, since it does not depend on sky brightness:

```
D_res = (138 / θ) × P(Δm)
```

θ = separation in arcseconds. P(Δm) is a penalty factor for unequal-brightness pairs: 1.0 for Δm ≤ 1.5, rising linearly to a cap of 1.5 at Δm ≥ 4. If D_res exceeds 150 mm, the pair is unresolvable at any sky and mBortle = Err regardless of brightness.

5.3 Module 2 — Extended objects (globular clusters, large planetary nebulae, galaxies)

```
μ_bg = μ_sky(b) + 0.242
C = 10^[−0.4 · (μ_obj − μ_sky(b))]
C_inf = 0.05 × 10^[0.2 · (μ_bg − 21.0)]
θ_R = 40′ × 10^[0.1 · (μ_bg − 21.0)]
Detected if:  C ≥ F × C_inf × (1 + θ_R / θ_app)²
```

μ_obj = the object's catalog surface brightness (mag/arcsec²). θ_app = 25 × object size (arcmin) — the apparent size at 25× magnification, which is the exit-pupil-optimal magnification for a 150 mm scope with a 6 mm eye pupil (M = D / 6). 0.242 mag is the light loss from 80% optical transmission. C is the object's contrast against the sky background; C_inf and θ_R are threshold-contrast parameters from the de Vries–Rose visual contrast model, which predicts that the eye's contrast threshold rises as background brightness falls. F = 2.0 is a safety margin on the threshold, within the empirically supported range from the source literature. mBortle is the highest b for which the detection inequality holds.

5.4 Module 3 — Dark nebulae

```
A_V: opacity 1→0.5, 2→1.0, 3→1.5, 4→2.5, 5→4.0, 6→6.0 (mag)
f = 10^(−0.4 · A_V);   B_stars = 10^(−0.4·μ_field);   B_sky(b) = 10^(−0.4·μ_sky(b))
C = (1 − f) · B_stars / (B_stars + B_sky(b))
```

A_V is the extinction of the Lynds opacity class (parsed from the source "Opacity=N" field). μ_field = 21.5 mag/arcsec² is the frozen brightness of the Milky Way star-cloud background the nebula silhouettes against. C is the resulting contrast, diluted by sky glow; the same threshold test from Module 2 is then applied using this C. Because a silhouette needs a visible backdrop, the sweep for dark nebulae is capped at Bortle 4 — classes brighter than 4 are not evaluated, since the Milky Way itself is not usable as a backdrop there.

6. Assigned and Assumed Values

Constant	Value	Basis
Aperture (D)	150 mm	Fixed 6-inch telescope, by design
Eye pupil	6.0 mm	Average dark-adapted observer
Magnification	25× (= D/6)	Exit-pupil optimum, pinned
Optical transmission	0.80 (→ 0.242 mag)	Typical multi-element eyepiece/scope losses
Altitude penalty	0.15 mag	45° typical target altitude
Aperture margin	×1.2 (→ 0.396 mag)	Safety margin on point-source detection
Contrast margin (F)	2.0	Safety margin on extended-object/DN threshold
Bortle 4 anchor	21.0 mag/arcsec²	Matches the companion aperture tool for consistency
Milky Way field (M3)	21.5 mag/arcsec²	Representative bright star-cloud background
DN visibility cap	Bortle ≤ 4	Milky Way must be visible as a backdrop

7. Source Resources

The Bortle-class-to-sky-brightness table, contrast-threshold model, and point-source limiting-magnitude behavior are derived from published amateur- and research-astronomy references, including:

- Bortle, J. E. (2001). "Gauging Light Pollution: The Bortle Dark-Sky Scale." Sky & Telescope — origin of the qualitative 1–9 class scale.
- Crumey, A. (2014). "Human contrast threshold and astronomical visibility." Monthly Notices of the Royal Astronomical Society, 442(3), 2600–2619 — source of the contrast-threshold (de Vries–Rose) equations and the finding that telescopic point-source limits are only weakly sky-dependent.
- Blackwell, H. R. (1946) — foundational visual contrast-threshold data underlying the detection model.
- Schaefer, B. E. (1990). "Telescopic Limiting Magnitudes." Publications of the Astronomical Society of the Pacific, 102, 212.
- Clark, R. N. (1990). Visual Astronomy of the Deep Sky — observational detection thresholds and reference tables.
- Commonly cited Sky Quality Meter (SQM) brightness ranges associated with each Bortle class, used to build the frozen µ_sky lookup table.

These sources were consulted during tool development to derive and validate the frozen constants; the tool itself does not perform live web lookups at run time — all values are pre-computed and hard-coded for reproducibility.

8. Data Integrity and Source-Data Logic

8.1 Validation of the model

The frozen constants and equations were checked against well-known reference objects with established observing difficulty (e.g., a bright globular cluster expected to be visible even from a city sky; a low-surface-brightness galaxy known to require very dark skies) to confirm the outputs match real-world observing experience before being applied to catalog data.

8.2 Handling of source spreadsheet data

- All values are parsed defensively (numbers may be stored as text); non-numeric or unparsable values are treated as missing, never guessed.
- Known placeholder values (Magnitude 2 = 99, 99.0, or 0) are recognized and excluded rather than treated as real data.
- If a required field for an object's routed module is missing (e.g., no magnitude, no size, no opacity class), the tool never substitutes an assumed or externally-sourced value — it returns "Err" and states the specific missing field.
- The double-star resolution requirement is evaluated before any sky-brightness test, since geometry alone can rule out detection regardless of sky quality; this keeps a physically impossible case from ever being reported as sky-dependent.
- Only the designated output column is modified. Other columns — including a separate minimum-aperture result from a companion tool — are never read from or written to, preventing cross-contamination between tools.
- Every "Err" result is visually flagged (yellow fill) and documented with a comment stating the exact cause, so a reviewer can distinguish "the physics says this is not observable" from "the source data is incomplete" at a glance, and correct the source data rather than the output.
- Because two uploads with similar file names have been found to contain different underlying data during this project, the tool always re-reads and recomputes from the actual current file contents rather than assuming consistency based on a filename.